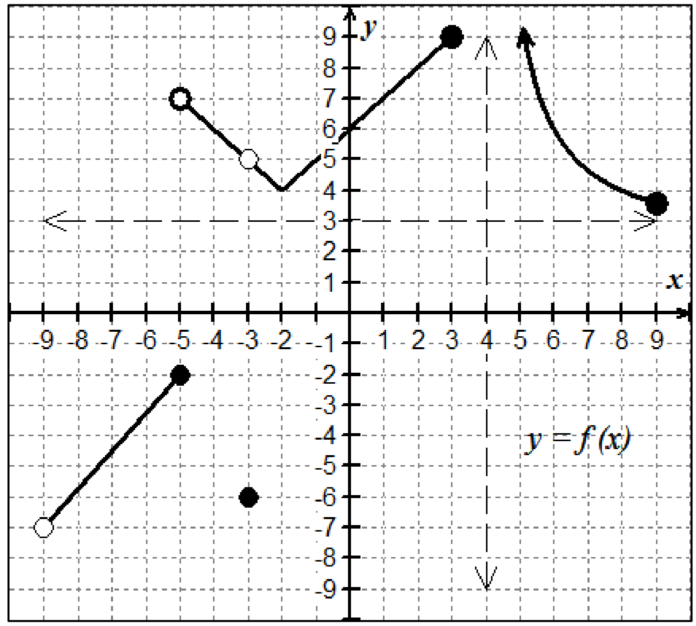
Limits and Continuity

Use the graph below to complete the table with the given information. After filling in the table, you should be able to write the three pieces of information that must be true in order for a function, $F(x)$, to be continuous at $x = a$.



1.

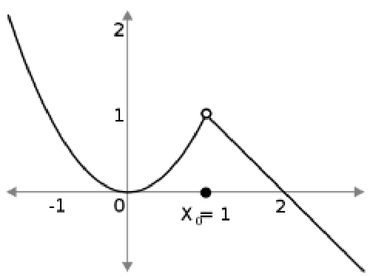
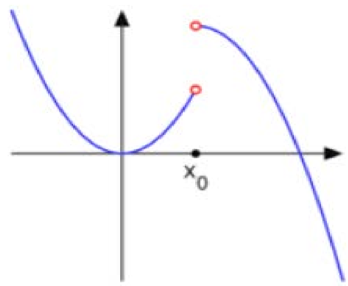
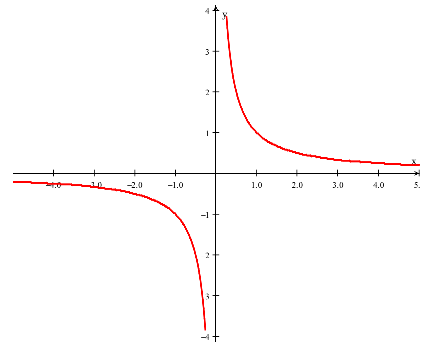
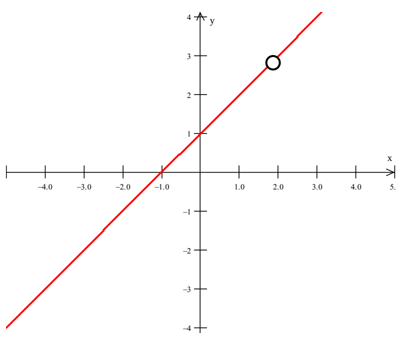
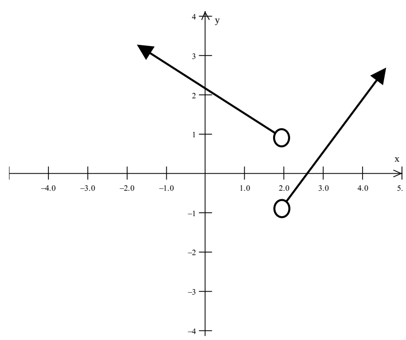
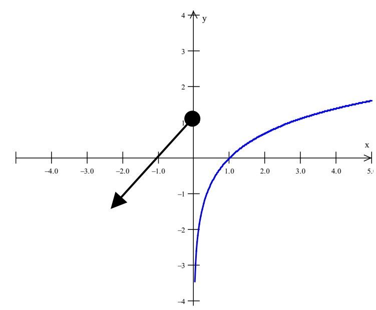
2.

3.

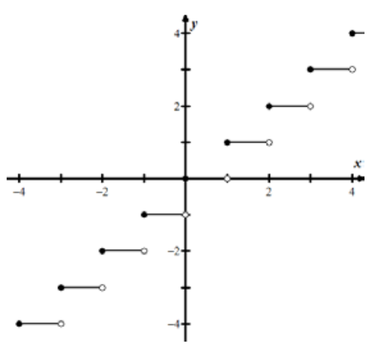
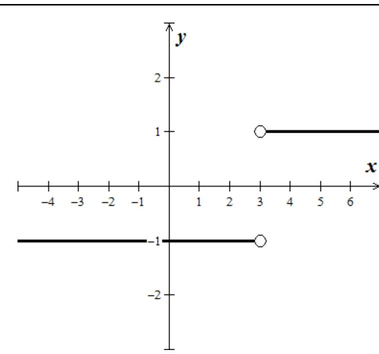
$x = a$	Is the function defined? If so, what is its value?	What is the value of $\lim_{x \to a^-} F(x)$?	What is the value of $\lim_{x \to a^+} F(x)$?	What is the value of $\lim_{x \to a} F(x)$?	Is $F(x)$ continuous at $x = a$?
$x = 9$					
$x = 4$					
$x = 3$					
$x = 0$					
$x = -3$					
$x = -5$					

*While **limits** told us where a function intended to go.*
***Continuity** guarantees that the function actually made it there.*

Classifying Discontinuities

Removable or Point (Holes) **2-sided limit exists**	**Non-Removable**	
	Jump 1-sided limits exists	**Infinite** At least one of the 1-sided limits doesn't exist
		
		

EX #1: Discuss the continuity of the following:

A. $f(x) = [x]$ 	B. $g(x) = \dfrac{\lvert x-3 \rvert}{x-3}$ 

Definition of Continuity – More Facts and Theorems

Continuity on an Open Interval:

A function is continuous on an open interval $(a\ , b\ )$ if it is continuous at each point in the interval.

Continuity on a Closed Interval:

A function is continuous on a closed interval $[a, b\ ]$ if it is continuous on the open interval $(a\ , b\ )$ and the function is continuous from the right at a and continuous from the left at b.

Properties of Continuity:

Given functions f and g continuous at $x = c$, then the following functions are also continuous at $x = c$.

1. Scalar multiple: $b \cdot f$
2. Sum or difference: $f \pm g$
3. Product: $f \cdot g$
4. Quotient: $\frac{f}{g}$; if $g(c) \neq 0$
5. Compositions: If g is continuous at c and f is continuous at $g\ (c\ )$, then the composite function is continuous at c, $(f \circ g)(x) = f(g(x))$

Continuity Laws of Some Basic Functions:

- Polynomial functions $P(x)$ are continuous over reals
- Rational Functions $P(x)/Q(x)$ is continuous on its domain such that $Q(c) \neq 0$.
- $y = x^{1/n}$ is continuous on all reals if n is odd and continuous on $[0, \infty)$ if n is even.
- $y = \sin x$ and $y = \cos x$
- $y = b^x$ is continuous for $b > 0, b \neq 1$
- $y = \log_b x$ is continuous for $x > 0, b > 0, b \neq 1$
- Inverse functions - if $f(x)$ is continuous on an interval with range R and $f^{-1}(x)$ exists, then $f^{-1}(x)$ is continuous on domain *R.*

One-Sided Continuity: A function $f(x)$ is called:

- **Left-continuous** at $x = c$ if $\lim_{x \to c^-} f(x) = f(x)$
- **Right-continuous** at $x = c$ if $\lim_{x \to c^+} f(x) = f(c)$

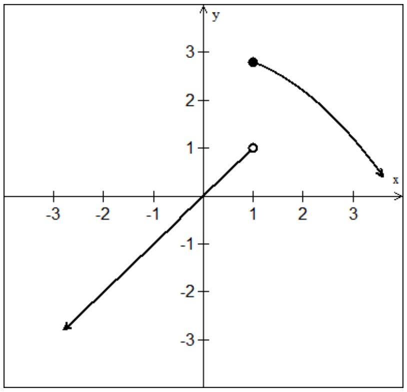
Continuity at a Point:

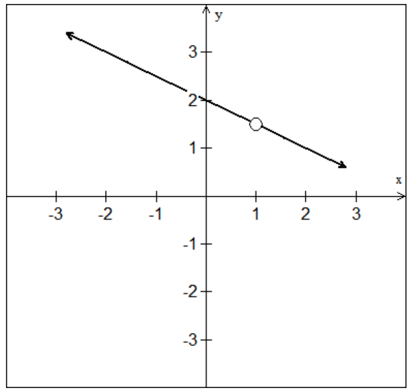
Suppose $f(x)$ is defined on an open interval containing $x = c$.
Then f is continuous at $x = c$ if

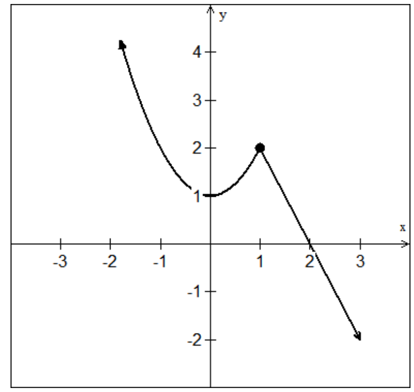
$$\lim_{x \to c} f(x) = f(c)$$

Other Facts:

EX #2: Continuous or Not? Defend your answer with a condition from the three point definition of continuity.







EX #3: Without graphing, justify why the functions are continuous for all real numbers.

A. $y = x^{\frac{2}{3}}$	**B.** $y = e^x$
C. $f(x) = 2x - \cos x$	**D.** $g(x) = \frac{1}{x^2 + 1}$

EX #4: If the function f is continuous and if $f(x) = \frac{x^2-4}{x+2}$ when $x \neq -2$, then $f(-2) = ?$

EX #5: For what value(s) of the constant c is the function g continuous over all the reals?

$$g(x) = \begin{cases} cx + 1\,; & x \leq 3 \\ cx^2 - 1\,; & x > 3 \end{cases}$$

EX #6: Given $h(x) = \begin{cases} -2x - 5\,; & x < -2 \\ 3\,; & x = -2 \\ x^3 - 6x + 3; & x > -2 \end{cases}$ for what values of x is $h(x)$ not continuous? Justify.